"On a Direct Method of Measuring the Coefficient of Volume Elasticity of Metals." By A. MALLOCK, F.R.S. Received April 19,—Read June 2, 1904.

For most hard materials the coefficient of volume elasticity is usually calculated from measurements of Young's modulus or of the coefficient of rigidity, either of which, when Poisson's ratio is known, suffices for its determination. Although, however, the total alteration of volume for a given pressure can be calculated from the coefficient thus obtained, it is only for isotropic material that the alteration of dimensions in any given direction can be inferred from it.

The following direct method of measuring the coefficient of volume elasticity is of some interest, as it allows of the linear contraction or extension in any given direction, caused in a substance by fluid pressure, to be measured independently of other changes of form.

When a long circular cylinder is acted on by internal fluid pressure, if the walls are very thin compared to the diameter of the cylinder, the stress in the material parallel to the axis is just half the stress at right angles to the axis in the tangent plane. The conditions of strain and stress in the walls of the cylinder can be expressed in terms of the volume elasticity and rigidity of the material as follows:—

Consider a small cube of the material, with edges parallel to the axis (X), radius (Y), and tangent of circular section (Z) of the cylinder respectively. Let K be that part of the stress which produces alteration of volume, and N_{xz} and N_{yz} the parts which produce shear in the planes XZ and YZ.

The total force acting parallel to Y, *i.e.*, in the direction of the radius, vanishes in comparison with the forces at right angles to it when the walls of the cylinder are thin, for while the radial force is O at one surface of the cylinder, and equal per unit area to the fluid pressure (P) on the other, in the two directions at right angles to the radius the stress is of the order Pr/t ($= 2P'$), where r is the radius and t the thickness of the wall.

Hence we have the following relations between K, N, and P':

$$K + N_{xz} + N_{yz} = 2P' \quad \ldots\ldots \quad (1),$$
$$K - N_{xz} = P' \quad \ldots\ldots \quad (2),$$
$$K - N_{yz} = O \quad \ldots\ldots \quad (3),$$

whence

$$K = P' \quad \ldots\ldots \quad (4).$$

That is, the alteration in the length of a cylinder caused by fluid pressure depends solely on the coefficient of volume elasticity.*

Since K is that part of the applied force which goes to produce alteration of volume, we have, if κ be the coefficient of volume

* It may be shown that this result is independent of the thickness of the walls of the circular cylindrical tube.

elasticity ($\kappa = \kappa_x + \kappa_y + \kappa_z$) and e_x, e_y, e_z, the alterations of dimensions caused in the cube by volume expansion in X, Y, Z,

$$\kappa_x = \frac{\mathrm{P}'}{e_x} = \mathrm{P}\frac{r}{2te_x}.$$

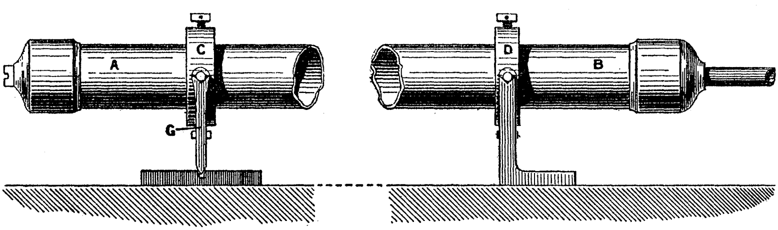
In isotropic materials $\kappa_x = \kappa_y = \kappa_z$ and $e_x = e_y = e_z$, so that

$$\kappa = \mathrm{P}r_2/6te.$$

The ease with which minute variations of length can be measured in the case of rods or tubes allows of a very accurate determination of that component of K which refers to stress parallel to the axis of the tube. For this purpose it is merely necèssary to so mount a suitable length of tube of the material to be experimented on that it can be subjected to strain by internal fluid pressure, and the variation of length caused by that pressure be observed.

I have used this method with steel, copper, and brass tubes in order to see whether annealing altered the value of κ. The arrangement employed is shown in fig. (1).

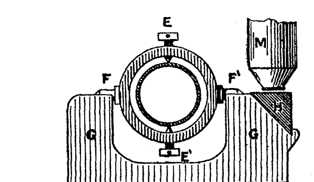
Fig. 1.



The tube AB is closed by caps at each end. Through the cap at B a small pipe connected with a pressure pump and gauge is introduced. The cap at A has a small hole in it, which can be closed with a plug after the tube has been filled with water.

The tube is embraced at C and D by gymbal rings fig. (2) where the steel points EE′ are closed on the tube, and the pivots FF′ rest at (D), fig. (1) on a fixed support, and at C, fig. (1) on a rocking support G. This rocking support carries a small reflecting prism H, whose upper surface is parallel to the axis of the tube, and passes through the axis FF′ of the gymbal ring. The movement of a mark on this surface of the prism was measured by the microscope M, fig. (2) a $\frac{1}{6}$-inch objective being used.

Fig. 2.



The only practical difficulty met with was the variation of temperature of the room during the experiments. From this cause it was generally found that the zero was slowly altering,

but the temperature effect can be readily eliminated from the observations.

In the experiments, the results of which are recorded below, the maximum fluid pressure in the tube was 400 lbs. per square inch. The longitudinal extension was measured at 100, 200, 300, and 400 lbs. pressure, and the extensions when corrected for temperature and plotted in terms of pressure lay on a straight line within the limits of error.

Table of Results.

Condition.	Length between supports in inches.	Outside diam. in inches.	Thickness of wall in inches.	Total extension at 400 lbs. per square inch.	$3\kappa_x$ in C.G.S. units.	κ from experiments.*
Seamless Steel Tube.						
Hard	60	0·75	0·0190	$2·8 \times 10^{-3}$	$18·4 \times 10^{11}$	$18·41 \times 10^{11}$
Annealed				2·75 ,,	18·2 ,,	
Solid Drawn Brass Tube.						
Hard	50	0·415	0·0185	$2·12 \times 10^{-3}$	1105×10^{11}	Different samples from $10·85 \times 10^{11}$ to 10·02
Annealed				2·09 ,,	1075 ,,	
Solid Drawn Copper Tube.						
Hard	50	0·4485	0·0382	$5·62 \times 10^{-4}$	23×10^{11}	$16·84 \times 10^{11}$
Annealed				7·1 ,,	16·2 ,,	

* From Everett's "C.G.S. System of Units."

The value of $3\kappa_x$ is given in the table to facilitate comparison with other measures, as $3\kappa_x$ is the value κ would have if the material had the same properties in all directions as it has in that of the axis of the tube.

The case of copper is remarkable, for in its hard drawn state it extends less in the direction of the drawing than a similar tube of steel would when equally strained.

FIG. 1.

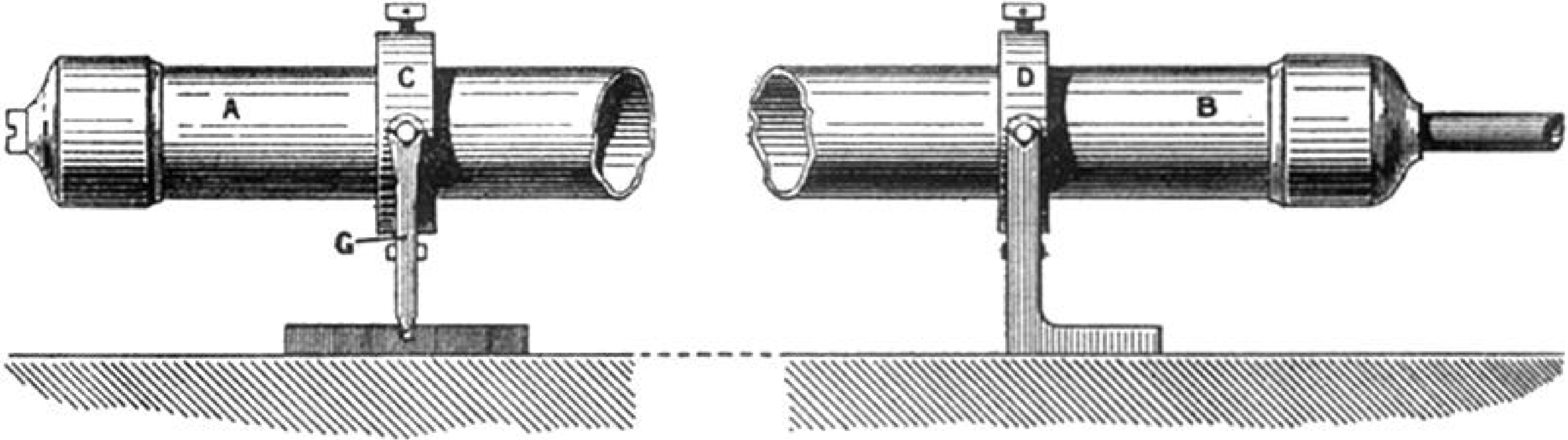


FIG. 2.